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On the control of rapidly rotating convection by an axially varying magnetic field

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The magnetic field in rapidly rotating dynamos is spatially inhomogeneous. The axial variation of the magnetic field is of particular importance because tall columnar vortices aligned with the rotation axis form at the onset of convection. The classical picture of magnetoconvection with constant or axially varying magnetic fields is that the Rayleigh number and wavenumber at onset decrease appreciably from their non-magnetic values. Nonlinear dynamo simulations show that the axial lengthscale of the self-generated azimuthal magnetic field becomes progressively smaller as we move towards a rapidly rotating regime. With a small-scale field, however, the magnetic control of convection is different from that in previous studies with a uniform or large-scale field. This study looks at the competing viscous and magnetic mode instabilities when the Ekman number E (ratio of viscous to Coriolis forces) is small. As the applied magnetic field strength (measured by the Elsasser number Λ) increases, the critical Rayleigh number for onset of convection initially increases in a viscous branch, reaches an apex where both viscous and magnetic instabilities co-exist, and then falls in the magnetic branch. The magnetic mode of onset is notable for its dramatic suppression of convection in the bulk of the fluid layer where the field is weak. The viscous–magnetic mode transition occurs at $\Lambda \sim 1$, which implies that small-scale convection can exist at field strengths higher than previously thought. In spherical shell dynamos with basal heating, convection near the tangent cylinder is likely to be in the magnetic mode. The wavenumber of convection is only slightly reduced by the self-generated magnetic field at $A \sim 1$, in agreement with previous planetary dynamo models. The back reaction of the magnetic field on the flow is, however, visible in the difference in kinetic helicity between cyclonic and anticyclonic vortices.

Keywords: Rapid rotation; Magnetoconvection; Two-scale convection; Geodynamo; Tangent cylinder; Helicity generation

1. Introduction

In planetary dynamos, convective motions are affected by the combined effects of rotation and the self-generated magnetic field. To improve our understanding of the dynamics in the nonlinear dynamo, the magnetoconvection problem, where convection in rapidly rotating systems is subject to an externally imposed magnetic field, is often studied. The simplest problem of this kind is given by a rotating horizontal fluid layer heated from below and permeated by a uniform magnetic field (Chandrasekhar 1961, Eltayeb 1972, Roberts and Jones

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2000). Following the theory of MAC waves (Braginsky 1967) that develop on the timescale of the geomagnetic secular variation, it was thought that the principal force balance in the Earth's core is between the Magnetic, Archimedean (buoyancy) and Coriolis forces. The magnetic Lorentz force aids convection in a rapidly rotating fluid by overcoming the Taylor-Proudman constraint (Eltayeb 1972), which in practice means that the critical Rayleigh number for onset of convection in a rotating fluid layer is smaller with a magnetic field than without a field. The idea that the magnetic field can break the rotational constraint and make convection possible implies that the ratio of the Lorentz to Coriolis forces, measured by the Elsasser number Λ , is of order unity.

In differentially heated spherical shells where all the heat enters the bottom boundary and there is no internal heating, the onset of rotating convection takes place near the tangent cylinder (TC), an imaginary cylinder touching the inner boundary and parallel to the axis of rotation (Dormy *et al.* 2004). Plane layer linear magnetoconvection with gravity g pointing in the axially downward (negative z) direction ignores the curvature of the polar regions, but is nevertheless a powerful tool in understanding TC magnetohydrodynamics. Convection in a rapidly rotating magnetic layer sets in either as narrow viscous columns or large-scale magnetic modes that fill a large fraction of the polar region (Chandrasekhar 1961), and these two patterns of instability are clearly visible within the TC in nonlinear dynamo simulations at different Rayleigh numbers (Sreenivasan and Jones 2005, 2006).

The case for using plane layer models to study spherical shell dynamo convection outside the TC is less straightforward. With the sloping boundaries preventing perfect geostrophy and the gravity vector pointing radially inward, spherical models differ geometrically and dynamically from plane-layer models. In addition, the magnetic Hartmann layer on the spherical boundary drives instabilities in an otherwise quiescent fluid (Zhang and Busse 1995), an effect not considered in plane-layer models. Nevertheless, there is a broad consensus on the effect of the magnetic field on the lengthscale of convection: spherical models predict that the main dynamical effect of the magnetic field is to reduce the wavenumber of convection (Longbottom et al. 1995, Jones et al. 2003), in line with plane layer models (Roberts and Jones 2000, Stellmach and Hansen 2004). Early dynamo simulations suggest that the magnetic field may thicken fluid rolls outside the TC (Kono and Roberts 2002), but simulations closer to the rapidly rotating regimes of planetary cores (e.g. Sreenivasan 2010) show that the self-generated magnetic field does not have any perceivable effect on the wavenumber of convection. This discrepancy between linear magnetoconvection models and dynamo simulations has not been adequately addressed, and may well be due to spatial inhomogeneities in the dynamo magnetic field that are ignored in linear models for simplicity.

Because the shortest lengthscale of the onset solution in a spherical shell is in the azimuthal (ϕ) direction (Sreenivasan and Jones 2011), the ϕ -component of the magnetic field is expected to have the strongest effect on convection. The equatorially antisymmetric azimuthal magnetic field produced in dipole-dominated spherical dynamos is axially inhomogeneous. Rapid rotation confines the magnetic field into narrow regions either side of the equator, which can partly explain why the field does not change the wavenumber of convection outside the TC. At the same time, a small-scale magnetic field can in principle allow viscously controlled convection is the bulk of the fluid layer, and perhaps trigger magnetically controlled convection where the field is strong. In such a configuration, it is not clear what would be the nature of the back reaction of the magnetic field on the flow. Previous models of plane-layer magnetoconvection use horizontal fields that are either axially uniform or having lengthscales comparable to the depth of the fluid layer. Among these models, some are inviscid (e.g. Kuang and Roberts 1990, Tucker and Jones 1997, Marsenić and Sevcík 2010), and therefore only consider magnetically



Figure 1. (a) Schematic of plane layer rotating magnetoconvection produced by a constant adverse temperature gradient under a spatially varying horizontal magnetic field. (b) The profile of B_0 in the layer from equation (1) for decay length scales $\delta = 0.14$ (red line) and $\delta = 0.4$ (blue line). A large-scale magnetic field profile using $f(z) = z(1-z^2)$ (black line) is given for comparison.

driven instabilities. Among the viscous models, Roberts and Jones (2000) use a uniform magnetic field, whereas Stellmach and Hansen (2004) use a spiral staircase structure. As large-scale convection sets in under large-scale magnetic fields even at small Elsasser number Λ , one might assume that planetary cores comfortably operate in the magnetic mode of convection, where the Lorentz force directly breaks the Taylor-Proudman constraint. However, with small-scale magnetic fields it is possible that the viscous mode of convection persists up to $\Lambda \sim 1$ and eventually gives way to the magnetic mode.

In this paper, we use plane layer magnetoconvection models to examine the role of a smallscale magnetic field in rapidly rotating convection, where the ratio of viscous to Coriolis forces (given by the Ekman number) is small. The linear onset study is followed by moderately supercritical spherical shell dynamo simulations, where the structure of convection outside the tangent cylinder is investigated.

2. Rotating convection with an axially varying horizontal magnetic field

2.1. Problem set-up and governing equations

We consider an electrically conducting fluid in a plane layer of infinite horizontal extent with rotation about the *z*-axis. The imposed horizontal (x) field goes to zero at the bottom boundary and has a finite axial lengthscale (figure 1). The magnetic field is of the form

$$\boldsymbol{B}_0 = \mathcal{B}_0 f(z) \hat{\boldsymbol{x}}, \quad f(z) = z \exp\left(-z^2/\delta^2\right), \tag{1}$$

where \mathcal{B}_0 is a reference magnetic field strength and δ is the axial decay lengthscale of the magnetic field. The lengthscale δ takes on small values under rapid rotation, but this effect has been largely ignored in previous models of magnetoconvection. When intense azimuthal magnetic fields are confined to narrow regions, the nature of convective onset can differ significantly from that for a large-scale magnetic field permeating the fluid layer.

The magnetoconvective instability of the system is studied by considering small perturbations to the basic states of velocity, magnetic field, pressure and temperature:

$$U = u, \quad B = B_0 + b, \quad P = P_0 + p, \quad T = T_0 - \beta z + \theta,$$
 (2)

where β is the basic state temperature gradient across the fluid layer. To obtain the perturbation equations in dimensionless form, lengths are scaled by the layer thickness *d*, time is scaled by d^2/η where η is the magnetic diffusivity, temperature is scaled by βd , velocity is scaled by η/d and the magnetic field is scaled by the reference field, \mathcal{B}_0 . In the Boussinesq approximation, the following linearized magnetohydrodynamic (MHD) equations govern the system:

$$EPm^{-1}\frac{\partial \boldsymbol{u}}{\partial t} + \hat{\boldsymbol{z}} \times \boldsymbol{u} = -\nabla p + A \big[(\nabla \times \boldsymbol{B}_0) \times \boldsymbol{b} + (\nabla \times \boldsymbol{b}) \times \boldsymbol{B}_0 \big] + PmPr^{-1}RaE\theta \hat{\boldsymbol{z}} + E\nabla^2 \boldsymbol{u} , \qquad (3)$$

$$\frac{\partial \boldsymbol{b}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{u} \times \boldsymbol{B}_0) + \nabla^2 \boldsymbol{b}, \qquad (4)$$

$$\frac{\partial \theta}{\partial t} = \boldsymbol{u} \cdot \hat{\boldsymbol{z}} + P \boldsymbol{m} P \boldsymbol{r}^{-1} \nabla^2 \theta \,, \tag{5}$$

$$\nabla \cdot \boldsymbol{u} = 0, \tag{6}$$

$$\nabla \cdot \boldsymbol{b} = 0. \tag{7}$$

The dimensionless parameters in these equations are the Ekman number E, Rayleigh number Ra, Elsasser number Λ , Prandtl number Pr and magnetic Prandtl number Pm, which are defined as follows:

$$E = \frac{\nu}{2\Omega d^2}, \quad Ra = \frac{g\alpha\beta d^4}{\nu\kappa}, \quad \Lambda = \frac{\mathcal{B}_0^2}{2\Omega\rho\mu_0\eta}, \quad Pr = \frac{\nu}{\kappa}, \quad Pm = \frac{\nu}{\eta}, \quad (8)$$

where ν is the kinematic viscosity, ρ is the density, κ is the thermal diffusivity, α is the coefficient of thermal expansion, g is the gravitational acceleration, Ω is the angular velocity of background rotation and μ_0 is the magnetic permeability. The ratio $PmPr^{-1}$ is also called the Roberts number, q. By applying the operators ($\nabla \times$) and ($\nabla \times \nabla \times$) to the momentum equation (3) and ($\nabla \times$) to the induction equation (4) and taking the *z*-components of the equations, the behaviour of the five perturbation variables – velocity, vorticity, magnetic field, electric current density and temperature – can be obtained. Solutions are sought in the form of normal modes

$$[u_z, \omega_z, b_z, j_z, \theta](x, y, z, t) = [W(z), Z(z), B(z), J(z), \Theta(z)] \exp[i(k_x + k_y) + st], \quad (9)$$

where k_x and k_y are wave numbers in x and y directions and s is a complex frequency. The horizontal wavenumber is $k = \sqrt{k_x^2 + k_y^2}$. After introducing this solution into our set of equations (3)–(5), we obtain the following system of ordinary differential equations:

$$EPm^{-1}s(\nabla^{2}W) = -k^{2}qERa\Theta + E(D^{2} - k^{2})^{2}W - \Lambda f''(z)ik_{x}B + \Lambda f(z)ik_{x}(D^{2} - k^{2})B - DZ, \qquad (10)$$

$$EPm^{-1}sZ = E(D^2 - k^2)Z + DW + \Lambda f(z)ik_x J - \Lambda f'(z)ik_y B, \qquad (11)$$

$$sB = (D^2 - k^2)B + f(z)ik_xW,$$
 (12)

$$sJ = f(z)ik_x Z + f'(z)ik_y W + (D^2 - k^2)J, \qquad (13)$$

$$s\Theta = W + q(D^2 - k^2)\Theta, \qquad (14)$$

where D = d/dz.

2.2. Boundary conditions

The stability calculations in this paper are performed with stress-free conditions for the flow at the top and bottom of the fluid layer. Electromagnetic conditions are mixed (perfectly conducting at the bottom and insulating at the top). However, to show that the main results of this study are not influenced by the boundary conditions, a few calculations at low Ekman number are performed with no-slip as well as perfectly conducting/insulating conditions at both walls. As isothermal conditions are maintained for the basic state, the temperature perturbation vanishes at the top and bottom. The boundary conditions are implemented as follows:

$W = D^2 W = DZ = 0$	at	z = 0, 1	(stress-free),	(15)
W = DW = Z = 0	at	z = 0, 1	(no-slip),	(16)
B = DJ = 0	at	z = 0	(bottom perfectly conducting),	(17)
DB + kB = J = 0	at	z = 1	(top insulating),	(18)
B = DJ = 0	at	z = 0, 1	(both walls perfectly conducting),	(19)
$DB \pm kB = J = 0$	at	z = 0, 1	(both walls insulating),	(20)
$\Theta = 0$	at	z = 0, 1.		(21)

2.3. Method of solution and benchmark

The input parameters for the model are E, Λ , Pr and Pm. We examine magnetoconvection regimes for the parameters $E = 2.5 \times 10^{-4} - 10^{-8}$, $\Lambda = 0 - 12$, Pr = 1 and Pm = 1. With temporal derivatives in the governing equations, onset of stationary convection occurs for Im{s} = 0 and oscillatory onset is marked by Im{s} $\neq 0$ (Re{s} = 0 yields the marginal state in either case). For a range of given Rayleigh numbers Ra, a bisection algorithm is employed to obtain the complex eigenvalue s. The Rayleigh number corresponding to the marginal state gives the critical Rayleigh number Ra_c . The eigenvalue problem $AX = \lambda BX$, where $\lambda = s$ and $X = [W, Z, B, J, \Theta]$, is solved using Matlab. A Chebyshev spectral collocation method is employed to decompose the eigenfunctions along z.

As shown in figure 2, our code accurately reproduces the oscillatory (overstable) onset of convection for Roberts number exceeding unity (q = 2.3), previously obtained in plane layer magnetoconvection at moderately large Ekman number (Roberts and Jones 2000). Since the stability calculations in our study use q = 1 throughout, we obtain purely stationary onset of convection. The onset of oscillatory convection with spatially varying magnetic fields is not considered as part of this study.

2.4. Properties of magnetoconvection at low Ekman number

The main result of this study is that the onset of magnetoconvection at low Ekman number often takes place with two competing modes of instability that co-exist – a large wavenumber *viscous* mode and a small wavenumber *magnetic* mode. Crucially, this two-scale onset is absent at high Ekman numbers, which implies that it is uniquely relevant to rapidly rotating regimes where the viscous force is small in comparison with the Coriolis force. We begin by looking at the Ra_c - Λ regime diagram for $E = 10^{-8}$, Pm = Pr = 1 and a spatially varying magnetic field B_0 of small decay lengthscale, $\delta = 0.14$. From figure 3(a), we find that Ra_c decreases briefly with increasing Λ , then increases until an apex (marked 'O') is reached, and thereafter



Figure 2. Benchmarked neutral (Ra-k) curves for the onset of magnetoconvection with spatially uniform horizontal magnetic fields acting on a rotating plane layer. The boundaries are stress-free and treated as perfect conductors of electric current. The black curve with the dashed branch shows stationary onset at k = 3.12, whereas the blue branch shows oscillatory onset at k = 4.8 for the same set of parameters ($E = 10^{-3}$, $\Lambda = 2$, $q = PmPr^{-1} = 2.3$ and $k_y = 0$). The red curve shows purely stationary onset at k = 2.498 for the parameters $E = 10^{-3}$, $\Lambda = 0.7$, q = 1 and $k_y = 0$.

decreases continuously over a range of Λ . Figure 3(a) shows the limited range $\Lambda = 0 - 1.5$ for clarity, but the critical parameters (Ra_c , k_c) up to $\Lambda = 12$ are listed in table 1. Figures 3(b–f) show the neutral (Ra–k) curves at points P and A on the rising branch, at the apex O, and at points C and Q in the falling branch. At point P, convection sets in at a large wavenumber that is not very different from the critical wavenumber for non-magnetic convection, which means that the lengthscale of convective rolls is controlled by viscosity. At point A ($\Lambda = 0.45$), a well-defined magnetic mode exists at small wavenumber, although the viscous mode is still dominant (figure 3(c)). At the apex O ($\Lambda = 0.545$), both viscous and magnetic modes have exactly the same Ra_c , and therefore must co-exist as equally unstable modes (figure 3(d)). At point C ($\Lambda = 0.65$), the magnetic mode has overtaken the viscous mode as the most unstable (figure 3(e)). The dominance of the magnetic mode instability continues to hold at point Q (figure 3(f)). Table 1 presents the critical parameters at various points on the regime diagram for $E = 5 \times 10^{-6}$, which is similar in pattern to that for $E = 10^{-8}$.

The axial velocity u_z in the marginal state (figures 4(a–d)) undergoes dramatic transitions as we travel along the regime diagram. For classical non-magnetic convection (A = 0), neatly packed columns that extend from z = 0 to z = 1 are obtained. At point A on the rising branch, the critical wavenumber of the flow is large, but slightly smaller than that for non-magnetic convection (table 1). Convection is suppressed in the bottom region where the magnetic field is strong (figure 4(b)). The viscous-mode instability is clearly dominant, although the magnetic field affects the flow at onset. At the apex O, where both viscous and magnetic modes are equally unstable, the axial velocity is a linear superposition of two eigenfunction terms

$$u_{z} = A_{1}W_{1}(z)\exp(ik_{x1}x) + A_{2}W_{2}(z)\exp(ik_{x2}x),$$
(22)

where W_1 and W_2 are the eigenfunctions of z-velocity at the viscous and magnetic modes, and k_{x1} and k_{x2} are the respective critical x-wavenumbers. We choose $A_1 = A_2 = 1$ to obtain the flow in figure 4(c). The large-scale flow in a narrow region at the bottom (~20% of the fluid layer) is magnetic-controlled, whereas the small-scale flow at the top (~80%) is viscous-controlled. Now, as Λ is increased slightly, the instability is entirely in the magnetic mode – in this regime, the small-scale flow is completely wiped out, leaving only the large-scale flow at the bottom (figure 4(d)). A horizontal magnetic field can therefore confine convection to



Figure 3. Plot (a) contains Ra_c vs. Λ for $E = 1 \times 10^{-8}$ and $\delta = 0.14$. Plots (b), (c), (d), (e) and (f) show the neutral curves extracted at points P, A, O, C and Q respectively on the regime diagram in (a).

a narrow region where the field is strong. It is remarkable that the change in flow structure from figures 4(b–d) takes place over a small range of Λ . From a comparative study of different Ekman numbers (section 2.5), it is clear that the suppression of large wavenumber convection by the magnetic field is a distinctly low Ekman number phenomenon. The axial velocity for the case $E = 5 \times 10^{-6}$ with $\delta = 0.3$ follows a very similar pattern, except that the critical wavenumbers are larger (table 1) and the large-scale flow in the magnetic mode extends up to $\sim 40\%$ of the fluid layer from the bottom (figures 5(a–d)).

A note on the robustness of the twin-mode instability is appropriate at this point. Figures 6(a,b) compare three electromagnetic boundary conditions, with the flow boundary conditions

$E = 10^{-8} \\ (\delta = 0.14)$			$E = 5 \times 10^{-6}$ ($\delta = 0.3$)			
Λ	Ra _c	k _c	Λ	Ra_{c}	k _c	
0	4.0363×10^{11}	605.6	0	1.0188×10^{8}	76	
0.1	3.0466×10^{11}	490	0.1	9.4095×10^{7}	67	
0.3	3.2274×10^{11}	499	0.3	1.0009×10^{8}	62	
0.45 (A)	3.2890×10^{11}	501	0.5	1.1021×10^{8}	63	
0.545 (O)	3.3175×10^{11}	51.1; 502.7	0.59	1.1352×10^{8}	63.5	
0.65 (C)	2.9744×10^{11}	46	0.69 (O)	1.1663×10^{8}	22.8; 64.2	
0.8	2.6388×10^{11}	41	0.79	1.1063×10^{8}	21	
1.0	2.3477×10^{11}	37	1.0	9.9991×10^{7}	18	
1.3	2.0790×10^{11}	32	1.3	8.9961×10^{7}	15	
1.5	1.9608×10^{11}	30	1.5	8.5309×10^{7}	14	
2.0	1.7723×10^{11}	26	2.0	7.7753×10^{7}	12	
5.0	1.4879×10^{11}	17	5.0	6.5879×10^{7}	8	
6.4	1.4705×10^{11}	16	6.4	6.5510×10^{7}	7	
7.0	1.4685×10^{11}	15	7.0	6.5163×10^{7}	7	
7.12 ^a	1.4683×10^{11}	15	7.2	6.5136×10^{7}	7	
7.2	1.4684×10^{11}	15	7.24 ^a	6.5135×10^{7}	7	
7.4	1.4691×10^{11}	15	7.4	6.5146×10^{7}	7	
8.0	1.4729×10^{11}	14	8.0	6.5375×10^{7}	7	
10.0	1.4893×10^{11}	14	10.0	6.6316×10^{7}	6	
12.0	1.5098×10^{11}	13	12.0	6.7393×10^{7}	6	

Table 1. Rayleigh numbers (Ra_c) and wavenumbers (k_c) for marginal state (critical) convection, computed for Elsasser numbers (Λ) in the range 0–12.

Notes: The linear stability runs are performed at two Ekman numbers (*E*), for Pr = Pm = 1, and with axially varying horizontal magnetic fields of lengthscale δ (equation (1)). The letter O refers to the apex and A, C are points to the left and right of the apex in the Ra_c -A regime diagram (figure 3(a)). Viscous and magnetic modes of instability co-exist at point O.

^aAlso note the existence of an absolute minimum in Ra_c for both Ekman numbers.

maintained stress-free. The regime diagram for insulating boundaries deviates slightly from the other two, but the co-existence of viscous and magnetic modes (marked by apex formation) holds at the same Λ for all three conditions. Furthermore, the neutral curves at any Λ overlap with one another and show two minima, showing that the co-existence of two unstable modes is practically insensitive to whether the boundaries are insulating, perfectly conducting or a combination of both. Figures 6(c,d) compare no-slip and stress-free boundary conditions for the flow, with the electromagnetic boundary conditions kept mixed. Although the regime diagrams and neutral curves for the two conditions do not exactly overlap, the two-mode solution with a sharp transition between the modes is obtained. As the stress-free solutions are computationally less expensive than the no-slip solutions, it makes sense to use stress-free boundary conditions to study linear onset at low Ekman number.

The dependence of the marginal state on the magnetic Prandtl number Pm is shown in figure 7, for $E = 5 \times 10^{-6}$, $\delta = 0.3$ and $\Lambda = 1.4$. The neutral stability curve develops three minima as Pm is increased, but the lowest minimum that represents the stationary magnetic mode of onset remains unchanged in the range Pm = 1-5. For $Pm \ge 6$, however, oscillatory onset at a higher wavenumber is obtained.



Figure 4. Shaded contour plots of the axial velocity at four points on the $Ra_c - \Lambda$ regime diagram (figure 3(a)), for the fixed parameters $E = 10^{-8}$ and Pr = Pm = 1. A restricted range of x is chosen for clarity. (a) $\Lambda = 0$ (non-magnetic convection); (b) $\Lambda = 0.45$ (Point A); (c) $\Lambda = 0.545$ (Apex O); (d) $\Lambda = 0.65$ (Point C).

2.5. The role of magnetic field lengthscale in convection

We now look at how different lengthscales of the magnetic field affect the pattern of convection in a plane layer. For the lowest Ekman number ($E = 10^{-8}$), small field lengthscales ($\delta \sim$ 0.1 - 0.25) produce a regime diagram with the apex marking the transition from viscous to magnetic mode instability (figure 8(a)). For a small field lengthscale $\delta = 0.12$, the apex forms at $\Lambda \approx 1$, which is when the Lorentz and Coriolis forces are thought to be in approximate balance. The critical wavenumber k_c at the apex expectedly falls sharply from its large viscousmode value to a small magnetic-mode value (figure 8(b)). However, for larger field lengthscales $(\delta = 0.3, 0.5;$ shown in dashed blue and dashed red lines), Ra_c and k_c decrease appreciably at small Elsasser numbers ($\Lambda \sim 0.1$) and then decrease gradually. The fact that the entry to the magnetic mode at $\Lambda \approx 1$ happens only for small field lengthscales is in qualitative agreement with nonlinear dynamo simulations at low Ekman number and $\Lambda \approx 1$, where the azimuthal magnetic field naturally assumes a small-scale structure (section 3). For $E = 5 \times 10^{-6}$ (figures 8(c,d)), the solutions are similar to that for $E = 10^{-8}$, except for two differences: the apex solutions at $\Lambda \approx 1$ happen for larger field lengthscales; and apex formation takes Ra_c above its critical value for non-magnetic convection (shown by the black dotted line). It is therefore likely that the magnetic mode is displaced to a point at the right of the apex where Ra_c falls below its non-magnetic value. Dynamo simulations (section 3) suggest that large magnetic field lengthscales ($\delta \gtrsim 0.5$) are unlikely to form at this Ekman number.

For higher Ekman numbers ($E = 5 \times 10^{-5}$ and 2.5×10^{-4}), large-scale magnetic fields look physically reasonable. For $E = 5 \times 10^{-5}$, an apex solution is obtained for a decay lengthscale $\delta = 0.3$ (blue dashed lines in figures figures 9(a,b)); however, Ra_c for magnetic cases con-



Figure 5. Shaded contour plots of the axial velocity at four Elsasser numbers, for the fixed parameters $E = 5 \times 10^{-6}$ and Pr = Pm = 1. (a) $\Lambda = 0$ (non-magnetic convection); (b) $\Lambda = 0.59$; (c) $\Lambda = 0.69$ (Apex); (d) $\Lambda = 0.79$. The critical parameters (Ra_c , k_c) at these points are given in table 1.

sistently exceeds its non-magnetic value, which is unrealistic for rotating magnetoconvection (Chandrasekhar 1961, Eltayeb 1972). The behaviour at higher δ (0.5 – 0.7) is realistic in that both Ra_c and k_c decrease progressively with Λ , in good agreement with the classical picture of large- δ magnetoconvection where the transition from the viscous to magnetic modes of instability is smooth (Longbottom *et al.* 1995, Roberts and Jones 2000, Chuxin and Xiaocheng 2003). For large δ , convection fills the entire fluid layer in both modes of instability, unlike in the apex solutions where the magnetic mode wipes out convection in regions with weak magnetic field. For $E = 2.5 \times 10^{-4}$ (figures 9(c,d)), no apex solution is obtained even for small δ . The magnetic mode for large δ is once again marked by a progressive reduction of Ra_c and k_c with Λ . Nonlinear dynamo simulations at moderately high Ekman numbers (section 3) indicate that the lengthscales of the azimuthal magnetic field are naturally larger than at lower Ekman numbers.

Figure 10 provides a study of the effects of large-scale and small-scale horizontal magnetic fields on rapidly rotating convection ($E = 10^{-8}$). The non-magnetic cases (shown with black circle markers) provide the reference against which comparisons can be made. For a large magnetic field decay lengthscale ($\delta = 0.6$), the axial velocity is significantly greater than its non-magnetic value (figure 10(a)), an effect that is reflected in the dramatic increase in kinetic helicity over the non-magnetic case (figure 10(c)). A different large-scale field of the algebraic form $f(z) = z(1 - z^2)$ also produces a strong augmentation in kinetic helicity. However, the effect of decreasing δ on helicity generation has not been addressed, either in planar or in spherical geometry. To understand this effect, we consider the case with a small-scale magnetic field ($\delta = 0.14$) and look at the viscous, mixed and magnetic modes of instability separately



Figure 6. (a) $Ra_c - \Lambda$ regime diagram for $E = 10^{-8}$, $\delta = 0.14$ and stress-free boundaries, comparing three electromagnetic boundary conditions. Both boundaries perfectly conducting (red marker), both boundaries insulating (blue line), and mixed with bottom perfectly conducting and top insulating (black line). (b) Neutral curve at $\Lambda = 1$ of plot (a), comparing the same conditions (with respective line styles). (c) $Ra_c - \Lambda$ diagram comparing stress-free (dashed blue) and no-slip (solid blue) boundary conditions for the flow. Mixed electromagnetic conditions are applied. (d) Neutral curve at $\Lambda = 1.5$ of plot (c), comparing the same conditions.



Figure 7. Neutral stability curves for $E = 5 \times 10^{-6}$, Pr = 1, $\Lambda = 1.4$ and $\delta = 0.3$. The cases shown are (a) Pm = 1 (red), (b) Pm = 3 (dashed black) and (c) Pm = 5 (blue).

(figures 10(d-f)). For the viscous mode (blue) the velocity is zero in the bottom region where flow is suppressed; for the mixed mode (red) the velocity increases sharply in the bottom region where the flow picks up strength; and in the magnetic mode (green) the velocity drops to zero for much of the upper region as convection is suppressed in the top 80% of the fluid layer. The patterns of velocity (figure 10(d)) and vorticity (figure 10(e)) are reflected in the kinetic



Figure 8. (a) $Ra_c - \Lambda$ regime diagram for $E = 10^{-8}$ and different lengthscales of the imposed magnetic field, δ . Lengthscales (with line style in brackets): 0.12 (red), 0.16 (blue), 0.2 (black), 0.3 (dashed blue), 0.5 (dashed red). (b) Critical wavenumber diagram $(k_c - \Lambda)$ for the same cases as (a), with respective line styles. (c) $Ra_c - \Lambda$ diagram for $E = 5 \times 10^{-6}$. Lengthscales (with line style in brackets): 0.3 (red), 0.35 (blue), 0.4 (black), 0.5 (dashed blue), 0.6 (dashed red). (d) $k_c - \Lambda$ diagram for the same cases as (c), with respective line styles. The black dotted line in each plot shows the reference critical value for the non-magnetic case.

helicity (figure 10(f)), where it is clear that the small-scale magnetic field does not offer any advantage in terms of enhanced helicity generation over the non-magnetic case. We find later from dynamo simulations that the self-generated magnetic field affects the helicity distribution appreciably even when the field lengthscale is small.

We conclude our discussion of linear magnetoconvection by noting the existence of an 'absolute minimum' in the critical Rayleigh number, Ra_c . Some early studies (Eltayeb and Kumar 1977, Fearn 1979) have confirmed the existence of a minimum for Ra_c , which implies that above a critical value of the Elsasser number Λ , as the Lorentz force exceeds the Coriolis force by a significant margin, the magnetic field inhibits convection. However, studies in spherical geometry (Zhang and Jones 1994, Longbottom et al. 1995) with imposed magnetic fields that satisfy the appropriate boundary conditions show that there is no optimal state of magnetoconvection marked by a minimum for Ra_c . Rather, there is a monotonic decay of Ra_c with Λ . For $\Lambda \sim 10$, the imposed magnetic field itself becomes unstable and drives fluid motion; consequently, it is not possible to have an absolute minimum. The Earth is therefore likely to operate in a $\Lambda \sim 1$ regime where the magnetic field is stable (Zhang and Jones 1994). In our study, an absolute minimum exists for Ra_c as in previous plane-layer models, but at $\Lambda \approx 7$ (table 1). As the regime where the Lorentz and Coriolis forces are in approximate balance ($\Lambda \approx 1$) is far from this minimum, we conclude that the Earth's core where this balance is expected to hold operates in a regime where the magnetic field promotes convection.



Figure 9. (a) $Ra_c - \Lambda$ regime diagram for $E = 5 \times 10^{-5}$ and different lengthscales of the imposed magnetic field, δ . Lengthscales (with line style in brackets): 0.3 (dashed blue), 0.4 (red), 0.5 (black), 0.6 (blue), 0.7 (dashed red). (b) Critical wavenumber diagram $(k_c - \Lambda)$ for the same cases as (a), with respective line styles. (c) $Ra_c - \Lambda$ diagram for $E = 2.5 \times 10^{-4}$. Lengthscales (with line style in brackets): 0.4 (dashed blue), 0.5 (red), 0.6 (black), 0.7 (blue), 0.8 (magenta). (d) $k_c - \Lambda$ diagram for the same cases as (c), with respective line styles. The black dotted line in each plot shows the reference critical value for the non-magnetic case.



Figure 10. Eigenfunctions of the axial velocity (W), axial vorticity (Z) and axial helicity (H) plotted as a function of z, for $E = 1 \times 10^{-8}$ and q = 1. The magnetic field lengthscale, $\delta = 0.14$. Line styles for (a)–(c): Non-magnetic convection (black marker), magnetoconvection with field length scale $\delta = 0.6$ (red line), magnetoconvection with field variation of the form $f(z) = z(1 - z^2)$ (blue line). (d)–(f): Non-magnetic convection (black marker); blue, red and green lines represent eigenfunctions at points A,O,C (before, at and after the apex, marking the viscous, mixed and magnetic modes respectively) in the Ra_c – Λ regime diagram (figure 3(a), obtained for $\delta = 0.14$).

E	Ra	Ra/Ra_c	Rm	E_m/E_k	Λ	E_{AD}	\bar{l}_B	$\delta(s = 0.54)$
2.5×10^{-4}	95.0	5	87.45	3.57	2.25	55.2	6.59	0.52
5×10^{-5}	115.0	4	126.86	5.93	1.55	68.6	9.24	0.44
5×10^{-6}	192.5	3.3	255.18	12.56	1.39	53.6	16.76	0.3

Table 2. Dimensional parameters used and magnetic field diagnostics computed in the dynamo models.

Notes: The parameter definitions are as in (8) except that the Rayleigh number $Ra = g\alpha\Delta T d/2\nu\kappa$ is the product of the classical Rayleigh number and the Ekman number, and the computed Elsasser number $\Lambda = B^2/2\Omega\rho\mu_0\eta$ is based on the volume-averaged magnetic field. The other fixed internal parameters are Pr = 1 and Pm = 3. Ra_c is the critical Rayleigh number for convection, Rm is the magnetic Reynolds number, E_m is the total magnetic energy, E_k is the total kinetic energy, E_{AD} is the ratio (in percent) of the axial dipole magnetic field (derived from the magnetic energy at the outer boundary, \bar{I}_B is the characteristic wavenumber of the magnetic field (derived from the magnetic energy spectrum) and δ is the axial lengthscale of the self-generated azimuthal magnetic field.

3. Nonlinear dynamo simulations

As the magnetic field in nonlinear dynamo simulations is self-generated, producing magnetic fields of Elsasser number $\Lambda \sim 1$ with marginally supercritical convection necessitates a large magnetic Prandtl number Pm, so that the magnetic diffusivity η is smaller than the viscous diffusivity ν (see, e.g. Willis *et al.* 2007). Here we essentially compensate for the weak energy input by allowing a relatively large electrical conductivity for the fluid. While setting up linear magnetoconvection at large Λ and Pm = 1 (as in table 1), we must realize that obtaining such strongly magnetic regimes at $Ra/Ra_c \approx 1$ in dynamo models requires large Pm. In this study, we consider three models: the Ekman number varies by a factor of 50 from maximum to minimum, the Rayleigh number is moderately supercritical so that the dynamical behaviour does not depart significantly from that at onset, and the choice of Pm = 3 makes the Elsasser number $\Lambda \sim 1$ in all models (see table 2). We recall that, for $E = 5 \times 10^{-6}$ both Pm = 1 and Pm = 3 have the same marginal state solution at $\Lambda = 1.4$ (figure 7), so the dynamo regime at $E = 5 \times 10^{-6}$ and Pm = 3 is not qualitatively different from the magnetoconvection regime at the same Ekman number and Pm = 1.

The dynamo models solve the coupled magnetohydrodynamic (MHD) equations for momentum, magnetic induction and temperature (Sreenivasan *et al.* 2014) in a spherical shell of radius ratio 0.35. Convection is driven by a superadiabatic temperature difference between the inner and outer boundaries, where electrically insulating and no-slip conditions are also satisfied. By focusing on regions near and away from the TC, we seek to understand the effect of the self-generated magnetic field on convection.

Figure 11 shows plots of the azimuthal magnetic field B_{ϕ} and on time and azimuthal average for the three dynamo models. The equatorially antisymmetric structure of B_{ϕ} corresponds to a dominant axial dipole magnetic field, and is a robust feature of numerical dynamos in a large region of the parameter space (Sreenivasan and Jones 2011). (The fact that the magnetic field is dominated by the axial dipole is evident from the fraction of the magnetic energy at the outer boundary contained in the dipole; see table 2). Furthermore, B_{ϕ} appears confined to smaller axial lengthscales as the Ekman number *E* is decreased. Figure 12 shows the change in the axial lengthscale of B_{ϕ} with *E* on a cylindrical section of radius s = 0.54, just outside the TC. It is readily confirmed that lowering *E* results in a magnetic field with smaller axial lengthscale. Magnetic field stretching via differential rotation (quantified by $B_z \partial u_{\phi}/\partial z$ in cylindrical polar coordinates) has a very similar structure to that of B_{ϕ} , which implies that the Ω -effect is an important mechanism for azimuthal field generation at the TC (figure 12). Outside the TC, the



Figure 11. Meridional section plots averaged over longitude and time of the azimuthal magnetic field B_{ϕ} at different Ekman numbers. Sections 1 - 1' and 2 - 2' correspond to cylindrical radii s = 0.54 and s = 1.0.



Figure 12. Time-averaged cylindrical $(z - \phi)$ section plots of the azimuthal magnetic field B_{ϕ} (upper panel) and the magnetic field stretching $B_z \partial u_{\phi} / \partial z$ (lower panel) for two Ekman numbers. The field is shown at cylindrical radius s = 0.54, whereas the stretching term is shown at a radius that lies just outside the Ekman layer at the inner boundary.

field and the stretching term can be oppositely signed (Olson *et al.* 1999) or of the same sign if differential rotation is strong (Schrinner *et al.* 2012).

In figure 13 we visualize the structure of convection on two cylindrical sections – just outside the TC (s = 0.54) and s = 1. The axial kinetic energy density u_z^2 is studied as it offers more clarity in plotting than the axial velocity itself. The dynamo calculations are compared



Figure 13. Time-averaged cylindrical $(z - \phi)$ section plots of the axial kinetic energy density u_z^2 . For each Ekman number, three plots are shown from top to bottom: Non-magnetic convection (s = 0.54), dynamo (s = 0.54) and dynamo (s = 1.0). The cylindrical sections are marked in figure 11.

with non-magnetic calculations, where the field strength Λ is scaled down to a very small value and only the momentum and temperature equations are stepped forward in time. For $E = 5 \times 10^{-6}$, the magnetic (dynamo) and non-magnetic cases have different structures at s = 0.54 (figures 13(a,b)): The axial energy in the dynamo is concentrated in narrow patches either side of the equator, whereas the flow in the non-magnetic case fills a larger depth of the layer. The suppression of convection near the TC in the dynamo is reminiscent of the magnetic mode instability in the plane layer (figure 5(d)). The structure of convection is dependent on the axial lengthscale of the magnetic field – for higher Ekman number runs where the azimuthal field takes on larger lengthscales along z, convection progressively fills the fluid layer (figures 13(e,h)), so the difference between the magnetic and non-magnetic cases is small. Further radially outward from the TC (s = 1.0), convection fills the entire fluid layer at all Ekman numbers (figures 13(c,f,i)), practically unaffected by the magnetic field lengthscale. (The non-magnetic plots at s = 1 look very similar to the magnetic plots and hence are not shown).

Figures 13(a,b) (for $E = 5 \times 10^{-6}$) also show that the maximum axial kinetic energy for the dynamo is larger than that for the non-magnetic run, an effect that is not obvious at higher Ekman numbers. The ratio of the axial to the total kinetic energy, E_z/E_{tot} supports this picture: for the lowest Ekman number, the dynamo calculation has more energy channelled into the axial flow than the non-magnetic calculation (table 3), consistent with linear magnetoconvection theory that predicts enhanced z-energy in the limit of $E \rightarrow 0$ (Sreenivasan and Jones 2011).

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E	\bar{l}_u	E_z/E_{tot} (shell)	$\frac{E_z/E_{tot}}{(s=0.54)}$	$\frac{E_z/E_{tot}}{(s=1.0)}$
2.5×10^{-4}	8.57 (9.99)	0.26 (0.25)	0.25 (0.20)	0.31 (0.30)
5×10^{-5}	10.95 (12.98)	0.21 (0.20)	0.21 (0.15)	0.24 (0.24)
5×10^{-6}	20.98 (25.36)	0.23 (0.18)	0.23 (0.14)	0.27 (0.24)

Table 3. Computed characteristic wavenumber of convection, \bar{l}_u and the fraction of the total kinetic energy contained in the axial (z) mode.

Note: The non-magnetic values are given in brackets for comparison with the dynamo values.



Figure 14. Isosurface snapshots of the axial velocity u_z , with positive (red) and negative (blue) shown in separate panels. The parameters used are $E = 5 \times 10^{-6}$, Pr = 1, Pm = 3 and $Ra/Ra_c = 3.3$ (a) and (b): non-magnetic run with a contour level of 30 % of the peak values (\pm 823). (c)–(h): magnetic run with peak values (-719, 728). Contour levels for (c) and (d) are 25% peak, (e) and (f) 35% peak and (g) and (h) 45% peak.

However, closer examination reveals that the augmentation in E_z/E_{tot} for the dynamo occurs largely near the TC (s = 0.54), and futher radially outward (s = 1.0) the z-energy fractions in the magnetic and non-magnetic runs are approximately equal.

Table 3 also gives the characteristic wavenumber of convection \bar{l}_u for the dynamo and nonmagnetic runs, obtained as a weighted average from the kinetic energy spectrum (see e.g. Soderlund *et al.* 2012). For all Ekman numbers, the dynamo-generated magnetic field causes only a modest reduction in \bar{l}_u from its non-magnetic value, in agreement with recent planetary dynamo simulations (Soderlund *et al.* 2012). However, this result is at variance with cartesian dynamo models (Stellmach and Hansen 2004) that show an appreciable increase in the flow lengthscale even for small Elsasser numbers. It is possible that the transition from the viscous to the magnetic mode happens at higher Elsasser numbers in spherical shell dynamos, but this point needs further investigation with the help of spherical magnetoconvection models.

To understand the magnetic control of convection in a spherical shell dynamo, we look at volume plots of isosurfaces of the axial velocity, u_z . For non-magnetic convection, both positive (red) and negative (blue) velocities are evenly distributed over the volume (figures 14(a,b)). In contrast, the saturated state ($\Lambda \sim 1$) of the dynamo shows that positive velocities are favoured in the upper hemisphere, whereas negative velocities dominate in the lower hemisphere. The asymmetry between the two signs of velocity is marked over a range of contour levels (figures

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Figure 15. Time-averaged isosurfaces of axial helicity, with anticyclonic helicity in the upper panel and cyclonic helicity in the lower panel. The parameters used are $E = 5 \times 10^{-6}$, Pr = 1, Pm = 3 and $Ra/Ra_c = 3.3$. (a) and (b): non-magnetic state with contour levels $\pm 5 \times 10^5$. (c)–(h): saturated axial dipole state with contour levels $\pm 3 \times 10^5$ (c,d), $\pm 7 \times 10^5$ (e,f), $\pm 1 \times 10^6$ (g,h).

14(e–h)), and is attributed to the magnetic (M)–Coriolis (C) term interaction in the *z*-vorticity equation (Sreenivasan *et al.* 2014). The effect of the dipole magnetic field also shows up in the distribution of *z*-vorticity, with anticyclonic vorticity preferred over cyclonic vorticity. The asymmetry in velocity and vorticity are in phase, producing an amplified asymmetry in the kinetic helicity. Figure 15 shows the time-averaged volumetric distribution of axial kinetic helicity contained in anticyclones and cyclones in separate panels. For the non-magnetic run, both cyclonic and anticyclonic helicity are present in equal measure (figures 15(a,b)). However, in the saturated dipole state, there is a notable preference for anticyclonic helicity (figures 15(c–h)). In summary, a low-*E*, $\Lambda \sim 1$ dipolar dynamo has self-generated helicity, even as the magnetic field does not appreciably change the wavenumber of convection.

4. Concluding remarks

The magnetic field in rapidly rotating dynamos is inhomogeneous in nature. While radial (s) and azimuthal (ϕ) inhomogeneities in the magnetic field can modify the flows in nonlinear dynamos, the axial (z) variation is of particular importance in rapid rotation, since tall fluid columns form at onset of convection. The fact that the magnetic field in rapidly rotating dynamos is confined to narrow regions near the equator has largely motivated our study. In this paper, we consider plane-layer magnetoconvection under vertical gravity and horizontal magnetic fields of controllable axial lengthscale. The ability of plane layer models to reach very low Ekman numbers makes them useful in the study of MHD regimes unreachable by present-day dynamo models.

In the rapidly rotating (low-*E*) regime, a sharp transition is noted between the viscous and magnetic instability modes in the Ra_c - Λ regime diagram, and interestingly, the viscous mode can exist up to $\Lambda \sim 1$. The magnetic mode of instability is notable for its complete suppression of convection in the weak-field region, which indicates that columnar convection is supported

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only in the viscous mode. As rapid rotation naturally confines the azimuthal magnetic field to small lengthscales, the large-lengthscale linear solutions at low E, although mathematically admissible, may not be physically meaningful.

In the spherical dynamo simulation at $E = 5 \times 10^{-6}$ and $\Lambda \approx 1.4$, the region close to the TC (s = 0.54) is marked by suppression of convection and an increase in the axial flow intensity, both of which are suggestive of the magnetic mode of convection. Away from the TC, however, the dynamo field does not have any notable effect on the depth or intensity of convection. It is possible that the viscous-magnetic mode transition far from the TC takes place at higher Λ , but this issue awaits further investigation with spherical shell magnetoconvection models using realistic azimuthal field distributions.

Since the magnetic field does not significantly change the wavenumber of convection outside the TC in spherical shell dynamo simulations, one might mistakenly conclude that magnetic and non-magnetic convection are not much different in character. The action of a $\Lambda \sim 1$ dipolar magnetic field is to generate a vortical asymmetry in kinetic helicity via the Lorentz–Coriolis force balance. It is remarkable that this effect, noted in earlier studies at $E = 3 \times 10^{-4}$ (Sreenivasan *et al.* 2014), persists at $E = 5 \times 10^{-6}$ despite the shorter lengthscale of the magnetic field. As *E* is lowered further, we may expect the azimuthal field to be confined to even smaller patches above and below the equator; nevertheless, the magnetic back reaction on the global helicity distribution is likely to persist in the rapidly rotating regime. An exception to this mechanism would be at large Rayleigh number, in which case strong buoyancy inhibits vortex stretching by the Lorentz force, reducing the relative helicity generated by the magnetic field (Sreenivasan *et al.* 2014).

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